BIO STATISTICS COLLABORATION OF AUSTRALIA

Study Guide

Mathematical Background for Biostatistics (MBB)

Semester 1, 2021

Prepared by:

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Contents

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Welcome Letter

Dear Student,

Welcome to Mathematical Background for Biostatistics (MBB). In this unit, we will develop the basic mathematical background needed to understand the proofs and mathematical reasoning used in the detailed treatment of biostatistical methods in subsequent units. Completion of this unit will allow you to concentrate on the statistical concepts presented in the later units without being distracted by the detail of the mathematical techniques.

In light of the preparatory nature of the material, the primary sources are two mathematics textbooks. There is little requirement for reading beyond these works.

One topic worthy of mention is the role of computer algebra systems (CAS) in relation to this unit. These are computer programs capable of solving abstract mathematical problems and are accessible on a number of platforms including CAS calculators, specialised packages such as Maple and Mathematica and on websites such as http://www.wolframalpha.com. Such packages are able to solve many of the problems given in the textbook with little e ort or understanding on the part of the user. It is therefore important to understand that the purpose of setting exercises is to help you develop skills in mathematical reasoning through practising the calculations rather than just to get a correct answer by any means available. It is, of course, convenient and useful to use a CAS package to check your calculations but you should not allow this to become the focus.

Please don't hesitate to contact us if you are having problems with the unit material.

Murthy N Mittinty & Meghana Kulkarni
March 2021

Unit Background

This unit of study is offered throughout Australia through the Biostatistics Collaboration of Australia (BCA). It is available in distance learning mode only, to students enrolled in postgraduate degrees in biostatistics coordinated by the BCA.

The purpose of MBB is to prepare students with little training in mathematics to study statistics at an advanced level. Students who have studied mathematics or statistics at undergraduate level, or who have equivalent work experience, are exempted from this unit.

On completion of this unit you should be able to follow the mathematical demonstrations and proofs used in biostatistics at Masters degree level, and to understand the mathematics behind the statistical methods introduced. This will allow you to concentrate on statistical concepts in subsequent units of your BCA program, with confidence in your mathematics.

The use of eLearning (sometimes referred to as Canvas) is very important in this unit and provides a guide to the unit material. This is the forum used to generate discussion of the unit content, to answer questions and to ensure that students have a solid comprehension of the necessary concepts.

Unit Objectives

On completion of this unit, students should be able to:

(1) Manipulate general mathematical expressions and inequalities.

(2) Understand the notion of a limit and calculate simple limits.

(3) Understand the notion of the derivative and its applications, and calculate simple derivatives.

(4) Understand the notion of the integral and its applications, and calculate simple integrals.

(5) Manipulate and evaluate simple matrix expressions.

(6) Understand matrix concepts such as determinant, inverse, rank, orthogonal matrix, eigenvalues and eigenvectors.

(7) Appreciate the nature and importance of mathematical arguments.

Unit Content

The subject will consist of three modules. These will cover the topics of:

Module 1	Numbers, Functions and Limits
Module 2	Calculus
Module 3	Matrices

Module 1 will require approximately 2 weeks of study, Module 2 will require approximately 4 weeks of study and Module 3 will require approximately 5 weeks of study, with a week free after each module for the associated assignment. The work for each week consists of readings and exercises. The exercises are not assessed, but the assignment questions will be similar so the exercises will be useful practice. Material will be accessed through the textbooks, which are required reading. Additional material will be provided as required. Notes for each module will be provided on eLearning and will include the relevant text references, notes and exercises. Written material will be supplemented by discussion on eLearning.

Software

The computing in this unit does not require a statistical software package. However, graphs are an important tool for understanding mathematics, and we assume you have access to either Wolfram Alpha, Microsoft Excel, R or Stata and can use it for calculations and for graphing functions. The student resources page on the BCA web site provides self-teaching materials for Excel, R and Stata.
Wolfram Alpha is free and can be accessed at http://www.wolframalpha.com. R can be freely downloaded from CRan. If you require us to provide R code for any of the exercises please do not hesitate to send us an email.

Learning Resources

You **WILL NEED** a copy of both of the following textbooks, making sure you have the exact edition mentioned:

Anton H, Bivens I and Davis S
Calculus Early Transcendentals, 11th Edition (Wiley, 2015) ISBN 9781118883822

It is available via Wiley Direct: http://www.wileydirect.com.au/buy/calculus-early-transcendentals-11th-edition/

It is also available from university bookshops or online from fishpond.com.au or amazon.com.

Be sure you have the correct version: not single variable, not late transcendentals and not brief edition. The ISBN identifies the right one.

Anton H
Elementary Linear Algebra, 12th Edition ISBN 978-1-119-28236-5
(Hardcover); eText: 978-1-119-40672-3

or
Elementary Linear Algebra: Applications Version, 12th Australia and New Zealand Edition Howard Anton, Chris Rorres, Anton Kaul
ISBNs E-Text: 9781119670858, Hard Cover Textbook: 9781119666066

Note that the two versions listed above are exactly the same for the purposes of this course. The exercises set, the required readings and the page numbers have all been checked to be identical between the two versions.

It is available via Wiley Direct: https://www.wiley.com/en-us/Elementary+Linear+Algebra%3A+Applications+Version%2C+12th+Edition-p-9781119406723

It is also available from university bookshops or online from fishpond.com.au or amazon.com.

Please note: WileyPLUS is **not** an accessible resource (or required) for students in this course. There is no need to purchase these textbooks with WileyPLUS should it be offered.

Workload requirements

The expected workload for this unit is 10-12 hours per week on average, consisting of exercises, discussions posts, independent study and completion of assessment tasks.

Method of Delivery

Students will be provided with three modules, as outlined in the previous section. These modules will also be made available on eLearning. The unit assessments will be available on eLearning and will not be provided to students on an individual basis. Important announcements will also be placed on eLearning, so students should regularly monitor eLearning.

Communication should generally be via eLearning (unless of a personal/ confidential nature) as responses to questions and discussion of issues is of benefit to all students. eLearning is an integral component of the MBB unit as it hopefully reduces the isolation which can occur in distance learning. Students can post questions, ideas, suggestions and discussion on eLearning. The tutors will monitor and respond to all communication, however students are also encouraged to respond and take part in these communications.

Staff Roles

The academic co-ordinator (Murthy N Mittinty) will be responsible for the unit and will contribute to the discussion on eLearning and respond to content-related questions.

Method of communication with coordinator(s)

You can contact Murthy N Mittinty directly in relation to requests related to personal matters. Email is the preferred method of contact for this kind of requests, with email addresses is stated earlier in this Study Guide.

To facilitate timely responses to your enquiries, please include BCA MBB and the module in question, or general enquiry, in the subject field of all emails. For example, include one of the following in the subject line of your email:

Assignment <number> deadline

Assignment <number> submission

Assignment <number> content question

Module <number> question

Textbook question

Administrative question

We would like to note, that we strongly encourage questions regarding course content to be posted on eLearning to generate and facilitate discussion.

For more general enquiries regarding the BCA program, or if you are having trouble contacting the academic coordinators/staff please contact Executive Officer.

Assessment

The assessment for this unit will involve three written assignments.

Assignment 1 will cover Module 1 and will be worth 20%.

Assignment 2 will cover Module 2 and will be worth 40%.

Assignment 3 will cover Module 3 and will be worth 40%.

All assignments will be posted on eLearning two (module 1) or three (modules 2 and 3) weeks prior to the submission date. Model solutions/guides will be posted on eLearning after the submission date.

Individual feedback on assignments will be provided to each student. Students are expected to monitor eLearning for the posting of assignments, solutions and feedback. Email notifications and other channels of communication will not be used. You may submit neatly handwritten work, however please note that marks will potentially be lost if the solution cannot be understood by the markers due to unclear or illegible writing. This handwritten work should be scanned and collated into a single pdf le and submitted via the eLearning site. See the BCA Assessment Guide document for specific guidelines on acceptable standards for assessable work. Once you scan your document please check that the page turnover nicely follow the order and the scanning is not done in reverse order on odd and even pages. The instructors will generally avoid answering questions relating directly to the assessable material until after it has been submitted, but we encourage students to discuss the relevant parts of the notes among themselves, via eLearning. However explicit solutions to assessable exercises should not be posted for others to use, and each student's submitted work must be clearly their own, with anything derived from other students' discussion contributions clearly attributed to the source.

Examples and exercises are contained in each module to enable students to ascertain their level of understanding of various topics. These will not form part of the assessment for this unit.

The Unit Timetable below shows the due dates for the assignments and a guide to the pace at which students should progress through the unit material.

Unit Timetable

Semester 1, 2021 will commence on Monday 1st March.

Study Week	Week Commencing	Topic	Assessment
1	1 March	Module 1: Numbers and Functions	
2	8 March	Module 1: Limits	
	15 March		Assignment 1 Available
3	22 March	Module 2: Calculus 1	
4	29 March	Module 2: Calculus 2	Assignment 1 Due: Monday 5th April
5	5 April	**Mid Semester Break 1 week only**	
6	12 April	Module 2: Calculus 3	
7	19 April	Module 2: Calculus 4	
8	26 April	Module 3: Matrices and Determinants	
9	3 May	Module 3: Vector Spaces I	Assignment 2 Due: Monday 3 May
10	10 May	Module 3: Vector Spaces II	Assignment-3 Available
11	17 May	Module 3: Least Squares	
12	24 May	Module 3: Eigenvalues, Eigenvectors and Diagonalization	
	31 May		Assignment 3 Due: Monday 31 May

Late Submission of Assessments and Extension Procedure

The standard BCA policy for late penalties for submitted work is a 5% deduction from the earned mark for each day the assessment is late, upto a maximum of 10

days (including weekends and public holidays). Extensions are possible, but these need to be applied for (by email) as early as possible. The Unit Coordinator is not able to approve extensions beyond three days; for extensions beyond three days you need to apply to your home university, using their standard procedures.

Submission of assessments and academic honesty policy

You should submit all your assessment material via eLearning unless otherwise advised. The use of Turnitin for submitting assessment items has been instigated within unit sites. For more detail please see pages 3-5 the BCA Student Assessment Guide. The BCA pays great attention to academic honesty procedures. Please be sure to familiarise yourself with these procedures and policies at your university of enrolment. Links to these are available in the BCA Student Assessment Guide. When submitting assessments using Turnitin you will need to indicate your compliance with the plagiarism guidelines and policy at your university of enrolment before making the submission.

eLearning

The online learning package used by the BCA is called eLearning (sometimes referred to as Canvas). The BCA eLearning site will be accessed through the University of Sydney (USyd) server. The BCA online facilities are, however, independent of the policies and procedures of this university. You will have access to online help at the USyd ITS and eLearning Helpdesks. A guide to getting started in eLearning is posted in the Student Resources section on the BCA website.

Online learning will be one of the tools used to provide access to materials and solutions to exercises, and as a communication tool. Students are encouraged to post content-related questions in the Discussion facility in eLearning. You will receive any specific instructions on using the eLearning site this semester from the BCA Coordinating Office. There is also a Getting Started document available on the Student Resources page of the BCA website.

eLearning Helpdesk

For further assistance with eLearning, you can contact the eLearning Helpdesk at http://www.usyd.edu.au/elearning/student/trouble/email_us.php

Please note, if you have queries about the subject matter for MBB, you should contact the academic coordinators.

If you are experiencing difficulties getting help, please contact the BCA coordinating office on 02 9562 5076, or email bca@ctc.usyd.edu.au.

Week by Week Key Points

Week 1

(1) The definitions of natural numbers, integers, rational numbers and irrational numbers.

(2) Understand and be able to use inequality, set and interval notation.

(3) The definition and properties of the absolute value, including the triangle inequality.

(4) Understand coordinates and be able to plot graphs of simple equations.

(5) Know the definition of the slope of a linear equation and understand the notion of parallel lines.

(6) The definitions of a function and the domain and range of a function.

(7) Understand the properties of the linear, power, polynomial, exponential and logarithmic families of functions, and be able to solve polynomial equations, and those involving exponentials and logarithms.

(8) Know the definition of an inverse function, and be able to find and graph the inverse of an invertible function.

(9) Know the definition of the factorial and be able to calculate and manipulate factorial expressions.

Week 2

(1) Understand the concept of a limit, including one sided limits.

(2) The exponential of a number can be written as a limit.

(3) Know the properties of limits and be able to use them to compute limits.

Week 3

(1) The definitions of the average and instantaneous rate of change and the derivative of a function.

(2) Be able to find the equation of the tangent line to a function at a given point.

(3) Understand the concept of differentiability.

(4) Be able to calculate the derivative of polynomial functions.

(5) Understand and be able to use the product, quotient and chain rules for differentiation.

Week 4

(1) Be able to calculate the derivatives of logarithmic and exponential functions.

(2) Know the definitions of increasing, decreasing, concave up and down, inflection points, relative maxima and minima and stationary points.

(3) Understand and be able to use both the first and second derivative tests.

(4) Know the geometric implications of multiplicity.

(5) Be able to sketch the graphs of polynomial equations.

Week 6

(1) The definitions of a function of two or three variables.

(2) Understand the concept of level curves and contour maps.

(3) Be able to calculate the partial derivatives of functions of two or three variables.

(4) Be able to apply Newton's method for the solution of equations.

(5) Understand when Newton's method will not converge to the solution.

(6) Be able to develop simple functions into their Taylor series.

Week 7

(1) Understand both the rectangle method and antiderivative method for finding areas.

(2) Know the definition of the indefinite integral and its properties.

(3) Be able to calculate indefinite integrals by the method of substitution.

(4) Understand sigma notation for sums and its properties.

(5) Understand the definition of the area under a curve as a limit.

(6) Understand the concept and properties of the definite integral.

(7) The Fundamental Theorem of Calculus (part 1) allows the calculation of the definite integral of any function for which the antiderivative is known.

(8) The Fundamental Theorem of Calculus (part 2) allows the derivative of a definite integral to be calculated, if the integral has a variable upper limit, a constant lower limit and the integrand is continuous.

(9) Be able to use both parts of the Fundamental Theorem of Calculus to calculate integrals of anti-differentiable functions, and to differentiate integrals.

(10) Both parts of the fundamental theorem of calculus show that differentiation and integration are inverse processes.

(11) There are two different methods which can be used to evaluate definite integrals of the form int f(g(x))g'(x)dx. You should understand

the two methods, but are free to use whichever you are more comfortable with.

(12) Only if interested: Integration by parts allows the calculation of integrals of the form int f(x)g(x)dx that are not appropriate for integration by substitution. You should understand how to use integration by parts, for indefinite and definite integrals, and also how to decide whether f'(x)G(x) or g'(x)F(x) is easier to integrate.

(13) Be able to calculate improper integrals over infinite intervals and for integrands that have infinite discontinuities.

(14) Understand the definition of the gamma function and be able to evaluate simply expressions involving the gamma function.

(15) Be able to calculate double integrals over rectangular and nonrectangular regions, and also know the properties of double integrals.

Week 8

(1) Understand the ideas of a matrix and a vector.

(2) Know how to multiply matrices by scalars.

(3) Understand the idea of the transpose of a matrix and its properties.

(4) Understand the idea of the trace of a square matrix.

(5) Know how to add and subtract matrices from other matrices and understand that this can only be done if their dimensions are identical.

(6) Know how to multiply two matrices together, and understand that this can only be done if the number of columns of the first matrix is equal to the number of rows of the second matrix.

(7) Know the definitions of diagonal, triangular and symmetric matrices and understand their properties.

(8) Understand the idea of the inverse of a square matrix, and its properties.

(9) Understand the idea of an identity matrix and its properties.

(10) Know how to form the inverse of 2x2 matrices.

(11) Know how to calculate the determinant of a square matrix, and understand the properties of determinants.

Week 9

For Euclidean vector spaces understand the concepts of

(1) Norm.

(2) inner product.

(3) orthogonality.

(4) subspaces.

(5) linear independence.

(6) basis.

(7) dimension.

Week 10

(1) General vector spaces: what is the difference and the relationship with Euclidean vector spaces.

(2) Know how to calculate the projection of one vector on another.

(3) Understand the idea of the rank of a matrix and the relationship with its invertibility.

Week 11

(1) Understand that the Euclidean dot product is an example of an inner product over a vector space, however this is the only example we will consider.

(2) Be able to apply the Gram-Schmidt Process to nd an orthonormal basis for a given space.

(3) Understand the matrix form of the unique least squares solution.

(4) Understand the matrix form in which the general linear model can be written.

Week 12

(1) Understand the ideas of eigenvectors, eigenvalues, and the characteristic equation of a matrix, and know how to find the eigenvalues and eigenvectors of a given matrix.

(2) Understand what it means for a square matrix to be diagonalizable, and when this occurs.

(3) Know the definition of an orthogonal matrix and their properties.

(4) Understand the spectral decomposition theorem.

(5) Know how to represent a square symmetric matrix in terms of eigenvalues and eigenvectors.

(6) Understand the idea of a positive definite matrix.

Assignment Submission

You will need to submit assignments using the submission links in the Assignments folder on eLearning.

Assignments must be typeset in the word-processor of choice. However, the submitted document must be in PDF format as a single file.[1]

If you are using Microsoft Word and wish to convert a .doc or .docx le to PDF (.pdf), choose the relevant set of steps below.

On Windows (Word 2013 onwards):

File → Export → Create PDF/XPS → Format: [select PDF] →Publish

On Windows (prior to Word 2013):

File → Save as... → Save as type: [select PDF] → Save On Mac:

File → Save as... → Format: [select PDF] → Save

Should the above fail, there are a multitude of online converters available.

Identifying details (MBB assignment and number, and your name) must be inserted in the header or footer box so that they appear on every page. You must also include the page number and the total number of pages on each page of your assignment (e.g. Page 1 of 10).

All submissions should be labelled with MBB assignment and number, and your initials (e.g. MBB–assignment1–ABC).

To submit your assignment, you first need to complete the relevant assignment declaration in the Assignments folder on eLearning. This will then activate the relevant assignment submission link, which will allow you to upload your assignment.

Further instruction about how to submit assignments online can be found on the BCA Assessment Guide.

BCA Assessment Guide - MBB

You should read through the BCA Assessment Guide in the Student resources page on the BCA website for further information on the following topics (http://www.bca.edu.au/currentstudents.html#assessmentguide):

Guidelines for written work

Guidelines for submission of assignments and exams

[1] A major reason for this is that Microsoft Word le formats can render differently or not at all on different computers, especially for computers with different add-ons installed. Files that are .pdf format are readable on all systems with rare exceptions.

BCA policies and procedures, including the complaints policy Own Work guidelines: advice on use of internet sites

Feedback

Our feedback to you:

The types of feedback you can expect to receive in this unit are:

Formal individual feedback on submitted assignments

Responses to questions posted on Blackboard

Your feedback to us:

One of the formal ways students have to provide feedback on teaching and their learning experience is through the BCA student evaluations at the end of each unit. The feedback is anonymous and provides the BCA with evidence of aspects that students are satisfied with and areas for improvement.

Acknowledgements

We would like to acknowledge some sources of help that are not otherwise acknowledged in the material. We thank Rachel Quill, Melissa Humphries and Maurizio Manuguerra for the use of existing BCA material for MBB that they developed. We thank previous coordinators, prior to Rachel Quill and Maurizio Manuguerra for the use of existing BCA material for MBB.